

Знайти границі функцій:

$$1. \lim_{x \rightarrow -1} (2x^3 - 4x^2 - 3x + 5) = 2 \cdot (-1)^3 - 4 \cdot (-1)^2 - 3 \cdot (-1) + 5 = -2 - 4 + 3 + 5 = 2.$$

$$2. \lim_{x \rightarrow 2} \frac{x^3 + 2x - 5}{2x + 1} = \frac{2^3 + 2 \cdot 2 - 5}{2 \cdot 2 + 1} = \frac{8 + 4 - 5}{5} = \frac{7}{5}.$$

$$3. \lim_{x \rightarrow 2} \frac{2x^2 - 9x + 10}{x^2 + 3x - 10} = \lim_{x \rightarrow 2} \frac{2(x-2)\left(x - \frac{5}{2}\right)}{(x-2)(x+5)} = \lim_{x \rightarrow 2} \frac{2x-5}{x+5} = \frac{2 \cdot 2 - 5}{2 + 5} = -\frac{1}{7}.$$

$$4. \lim_{x \rightarrow \infty} \frac{3x^3 - 5x^2 + 2}{2x^3 + 5x^2 - x} = \lim_{x \rightarrow \infty} \frac{x^3 \left(3 - \frac{5}{x} + \frac{2}{x^2}\right)}{x^3 \left(2 + \frac{5}{x} - \frac{1}{x^2}\right)} = \frac{3}{2}.$$

$$5. \lim_{x \rightarrow \infty} \frac{3x^4 - 6x^2 + 2}{2x^2 - 4x - 3} = \lim_{x \rightarrow \infty} \frac{x^4 \left(3 - \frac{6}{x^2} + \frac{2}{x^4}\right)}{x^2 \left(2 - \frac{4}{x} - \frac{3}{x^2}\right)} = \frac{3 \cdot \infty}{2} = \infty.$$

$$6. \lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 2}{2x^3 + 5x^2 - x} = \lim_{x \rightarrow \infty} \frac{x^2 \left(3 - \frac{5}{x} + \frac{2}{x^2}\right)}{x^3 \left(2 + \frac{5}{x} - \frac{1}{x^2}\right)} = \frac{1}{\infty} = 0.$$

$$7. \lim_{x \rightarrow 0} \frac{x}{\sqrt{16+x} - 4} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{16+x} + 4)}{(\sqrt{16+x} - 4)(\sqrt{16+x} + 4)} = \lim_{x \rightarrow 0} \frac{x(\sqrt{16+x} + 4)}{16 + x - 16} =$$

$$\lim_{x \rightarrow 0} \frac{x(\sqrt{16+x} + 4)}{x} = \lim_{x \rightarrow 0} (\sqrt{16+x} + 4) = \sqrt{16+0} + 4 = 4 + 4 = 8.$$

$$8. \lim_{x \rightarrow 0} \frac{\sin 5x}{x} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot 5 = 5 \cdot \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 5 \cdot 1 = 5.$$

$$9. \lim_{x \rightarrow 0} \frac{\cos 2x - \cos 6x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow 0} \frac{-2 \sin \frac{2x-6x}{2} \sin \frac{2x+6x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin(-2x) \sin 4x}{x^2} =$$

$$= -2 \lim_{x \rightarrow 0} \frac{-\sin 2x \sin 4x}{x^2} = 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot \frac{\sin 4x}{4x} \cdot 8 = 2 \cdot 1 \cdot 1 \cdot 8 = 16.$$

$$10. \lim_{x \rightarrow \infty} \left(\frac{x-3}{x+3}\right)^{2x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+3-3-3}{x+3}\right)^{2x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+3-6}{x+3}\right)^{2x+1} = \lim_{x \rightarrow \infty} \left(1 - \frac{6}{x+3}\right)^{2x+1} =$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{x+3}{-6}}\right)^{\frac{-6}{x+3} \cdot (2x+1)} = e^{\lim_{x \rightarrow \infty} \left(-\frac{6}{x+3} \cdot (2x+1)\right)} = e^{\lim_{x \rightarrow \infty} \left(-\frac{12x+6}{x+3}\right)} = e^{-12}.$$

$$11. \lim_{x \rightarrow 0} (1+7x)^{\frac{1}{2x}} = \lim_{x \rightarrow 0} \left[\left((1+7x)^{\frac{1}{7x}} \right)^{7x} \right]^{\frac{1}{2x}} = \lim_{x \rightarrow 0} \left((1+7x)^{\frac{1}{7x}} \right)^{\frac{7}{2}} = e^{\lim_{x \rightarrow 0} \frac{7}{2}} = \frac{7}{2}.$$

Вправи

1. $\lim_{x \rightarrow -1} (x^3 + x^2 - x + 1).$

3. $\lim_{x \rightarrow -1} \frac{3x^2 + 7x - 1}{x^3 + 4x^2 - 2x + 1}.$

5. $\lim_{x \rightarrow 11} \frac{x^2 - 121}{x - 11}.$

7. $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}.$

9. $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 9x + 20}.$

11. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}.$

13. $\lim_{x \rightarrow \infty} \frac{2x^3 - 3x^2 + 1}{x^3 + 4x^2 + 2x}.$

15. $\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 4}{x^6 + 2x^2 + 3x}.$

17. $\lim_{x \rightarrow 0} \frac{\sin x + \sin 5x}{\sin 2x}.$

19. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x-3}.$

20. $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^{4x-3}.$

23. $\lim_{x \rightarrow 0} (1+5x)^{\frac{7}{x}}.$

2. $\lim_{x \rightarrow -2} (2x^2 - 5x + 1).$

4. $\lim_{x \rightarrow -2} \frac{2x^3 - 7x^2 + 5x - 2}{4x^2 + 6x - 8}.$

6. $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x + 6}.$

8. $\lim_{x \rightarrow 2} \frac{3x^2 - 8x + 4}{5x^2 - 14x + 8}.$

10. $\lim_{x \rightarrow 7} \frac{x-7}{2 - \sqrt{x-3}}.$

12. $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - 1}{x}.$

14. $\lim_{x \rightarrow \infty} \frac{x^5 - 2x^2 + 3}{3x^3 - 5}.$

16. $\lim_{x \rightarrow 0} \frac{\sin 5x}{\operatorname{tg} 7x}.$

18. $\lim_{x \rightarrow 0} \frac{\sin 4x - \sin x}{\operatorname{tg} x}.$

21. $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x-3} \right)^{3x+2}.$

22. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x-3}.$

24. $\lim_{x \rightarrow 0} (1-4x)^{\frac{7}{x}}.$